Automatic Control of Adverse Yaw in the Landing Environment using Optimal Control Theory

David Bischoff,* Robert Duffy,† and Howard Kaufman‡
Rensselaer Polytechnic Institute, Troy, N.Y.

Nomenclature

A = state matrix
B = control matrix
f_{ij} = feedback gains; i-control, j-disturbance
H = terminal penalty matrix
Q = state weighting matrix
q_{ii} = diagonal components of state weighting matrix
R = control weighting matrix
r_{ii} = diagonal elements of control weighting matrix
α_A = absolute angle of attack of zero lift line, (deg)
β = angle of side slip (deg)
p, q, r = roll, pitch and yaw rates, respectively
$\delta a, \delta r, \delta e$ = aileron, rudder and elevator controls, respectively
ϕ, θ, ψ = roll, pitch and yaw angles, respectively

Superscripts and Subscripts

o = initial condition
f = final condition
$'$ = transpose of the matrix
$*$ = optimal condition
·(dot) = time derivative

Theme

A PRACTICAL application of optimal control theory techniques was investigated through digital simulation of the equations of motion defining an aircraft's response to lateral control inputs in the landing approach. A linear regulator solution was used to construct an automatic control system which would restrain the adverse yaw generated during such maneuvers within acceptable limits. The optimal control determined from this approach acted in response to lateral-directional state variables and consisted of rudder applied opposite to sideslip, coupled with a reduction in the pilot's commanded lateral control. The resulting reduction in adverse yaw, 50% for a military attack aircraft, demonstrates the feasibility of optimal control techniques in designing control systems.

Actual flight tests of aircraft in the landing approach environment have shown these aircraft to exhibit excessive adverse yaw. The pilot workload required to execute late lateral line-up corrections during actual approaches has been attributed to these adverse yaw characteristics.[1] Adverse yaw, generally consisting of yawing moment due to roll rate and lateral control deflection, is not a parameter over which the designer has a great deal of control, once the airplane design has been established. The problem then is one of designing a control system to reduce the sideslip angle excursions below levels acceptable to the pilot, thereby reducing his workload in a very demanding environment.

Received August 13, 1973; Presented as Paper 73-861 at the AIAA Guidance and Control Conference Kay Biscayne, Fla., August 20-22, 1973; Synoptic received May 20, 1974; revision received June 20, 1974; Full paper available from AIAA Library, 750 Third Avenue, New York, N.Y. 10017. Price: Microfiche, $1.00; hard copy, $5.00. **Order must be accompanied by remittance.**

Index categories: Aircraft Handling, Stability, and Control; Aircraft Flight Operations.

*Graduate Student, Department of Aeronautical Engineering and Astronautics; presently Aerospace Engineer, U.S. Naval Air Test Center.

†Chairman, Aeronautical Engineering and Astronautics. Associate Fellow AIAA.

‡Associate Professor of Systems Engineering.

Contents

The state variable representation of the kinematical and dynamical equations of motion of an aircraft may be formulated as

$$\dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \tag{1}$$

where

$$\dot{\mathbf{x}}' = [D\beta, D\alpha, Dp, Dq, Dr, D\phi, D\Theta] \tag{2}$$

$$\mathbf{x}' = [\beta, \alpha, p, q, r, \phi, \Theta] \tag{3}$$

$$\mathbf{u}' = [\delta a, \delta r, \delta e] \tag{4}$$

A,B = state and control matrices, respectively, composed of inertia terms and stability derivatives.

The stability derivatives contained in these equations are functions of the aircraft's absolute angle of attack α_A and were expressed as such in the nonlinear simulations. These derivatives are linearized by restraining them at the value associated with the initial trimmed states.

Optimal Control-Linear Regulator: The determination of an optimal controller for a linear system which minimizes a quadratic performance index, J, may be stated as; -given and multi-input, multi-output dynamic system represented by the set of linear state equations

$$\dot{\mathbf{x}}(t) = A\mathbf{x}(t) + B\mathbf{u}(t) \tag{5}$$

determine the control function, $\mathbf{u}(t)$, which minimizes the performance index, J

$$J = 1/2\mathbf{x}'(t_f)H\mathbf{x}(t_f) + 1/2\int_{t=0}^{t=\infty}[\mathbf{x}'(t)Q\mathbf{x}(t) + u'(t)R\mathbf{u}(t)]dt \tag{6}$$

It can be shown[3] that the optimal control, $u^*(t)$, is of the constant gain feedback form

$$u^*(t) = -R^{-1}B'K\mathbf{x}'(t) \tag{7}$$

where the gain matrix K satisfies the matrix algebraic Riccati equation

$$0 = -KA - A'K - Q + KBR^{-1}B'K \tag{8}$$

It has been shown by Kalman that for a controllable system this solution yields a stable closed-loop system for R positive definite and Q positive semidefinite.

Results

Uncontrolled Motions: The adverse yaw test maneuver (measurement of sideslip angle excursion during rudder-fixed, full lateral control deflection rolls through 90° of bank) was simulated for a jet attack aircraft using both the nonlinear and linear equations of motion. It is observed[2] that the linearized equations of motion provide an excellent approximation to the nonlinear equations for all lateral-directional rates and angles up to the maximum values obtained. The longitudinal responses of angle of attack, pitch angle and pitch rate are poorly approximated by the linearized equations of motion in this predominantly lateral-directional maneuver. However, this does not present any special problems in the analysis.

Controlled Motions: The optimal control calculation procedure assumed that there are no bounds placed on the controls utilized in producing the desired dynamic response. This assumption, while reducing the complexity of the optimal control calculation, becomes invalid when attempting to actually implement the control, due to the physical control limits on the aircraft. In addition, designing a control system which would utilize either a large portion or all of the control deflection available could result in a hazardous flight condition in the event of a failure, or "hard over" in the controller. Thus the control deflections were restricted by means of a signal limter at the output of the feedback gain controller. The controller was limited to 20% of full deflection of the aileron and elevator control surfaces and to a maximum of 40% of full rudder deflection.

Optimal Control of the Jet Attack Aircraft: In determining an optimal controller for the jet attack aircraft, the complete state equations were utilized with the performance function chosen to minimize both angle of sideslip and angle of attack excursions, with a minimum use of the three controls. The performance function is

$$J = 1/2\int_0^\infty [q_{11}\beta^2 + q_{22}\alpha^2 + r_{11}\delta a^2 + r_{22}\delta r^2 + r_{33}\delta e^2]dt \tag{9}$$

where q_{ii}, and r_{ii} are weighting terms to be varied to obtain the desired response.

The state matrix, A, contains many terms which are functions of the initial states, thereby yielding a different Riccati solution for each initial condition. Trial solutions of the control equations for various initial conditions (banked turns and wings level flight) and simulation of the corresponding roll response resulted in the following simplified linear feedback of the state variables in order to implement the controls:

$$\delta a = f_{a_\beta}\beta + f_{a_p}p + f_{a_r}r + f_{a_\Phi}\phi \tag{10}$$

$$\delta r = f_{r_\beta}\beta + f_{r_p}p + f_{r_r}r + f_{r_\Phi}\phi \tag{11}$$

$$\delta e = f_{e_\alpha}\alpha + f_{e_q}q \tag{12}$$

Solutions to the Ricatti equation were obtained with various values of the state and control weighing matrices. The resulting control sequences were then simulated for adverse yaw test maneuver. The results of these simulations are presented in Table 1.

Effect of Varying the State and Control Weighting Matrices on Control of the Jet Attack Aircraft: As the state weighting matrix was increased, the resulting feedback gains were increased which correspondingly commanded larger control deflections. However, for q_{ii} above a value of 50, there was little reduction in the amount of adverse yaw generated. Increasing the control weighting matrix resulted in corresponding reduction in the magnitude of the control commanded. However, as the maximum magnitude of the control was reduced, so was its initial magnitude, resulting in a more sluggish response of the airplane. The adverse yaw was reduced by decreasing the value of r_{ii} below 1.0. However, the magnitude of the commanded control became excessive and was held in for longer than desired at the completion of the maneuver.

Table 1 Adverse yaw test maneuver

δr_{MAX}	q_{ii}	r_{ii}	β_{MAX} (deg)	$\beta_{\phi=90°}$ (deg)	$\phi_{t=1\ sec}$ (deg)
0	0	0	−21.2	−13.4	−50.5
10	1	1	−12.3	−9.2	−44.2
	10	1	−10.8	−8.8	−44.7
	50	1	−10.2	−8.3	−44.7
	100	1	−10.1	−8.1	−44.2
	50	0.1	−10.0	−8.0	−44.0
	50	10	−11.4	−9.2	−45.7
	50	25	−12.9	−9.8	−50.3
15	10	1	−8.2	−7.2	−45.3
	50	1	−7.0	−6.0	−44.2
	100	1	−6.8	−5.8	−44.2
	50	0.1	−6.6	−5.4	−44.0
	50	10	−9.2	−8.0	−46.3
	50	25	−11.9	−9.4	−47.5

Therefore, considering the small performance improvement for $q_{ii} > 50$ and the sluggish controller action for $r > 1$, the gains chosen to optimize the airplane's rolling performance were obtained for $q_{ii} = 50$ and $r_{ii} = 1$.

Table 1 also shows the effect of the amount of rudder chosen to control the yawing motion. For the optimum gains chosen, limiting the rudder control to ±10° resulted in a 38% reduction in sideslip angle from the uncontrolled adverse yaw test maneuver. Increasing the allowable rudder control to ±15° resulted in a similar reduction of 55% in sideslip from the uncontrolled response. This significantly larger decrease in adverse yaw warrants the increased controller action of ±15°.

Suboptimal Control of the Jet Attack Aircraft: Three suboptimal constraints of the optimal control for the jet attack aircraft discussed previously were investigated. The first of these consisted of deleting the lateral control feedback terms, thereby controlling the adverse yaw buildup with rudder action only. This configuration resulted in an increase in the measured adverse yaw from 6.0° to 7.6° or a 27% increase while the total sideslip excursion in the roll exhibited a 48% increase. The bank angle attained in one second increased from 44° to 51°. The lateral control, therefore, was of valuable assistance in reducing the adverse yaw in rolling maneuvers and should be considered for application in those cases where its use does not restrict the aircraft's rolling performance.

The other suboptimal controls were obtained by eliminating some of the states being fed back to the controller. Eliminating the bank angle feedback term, an 8.5% increase in measured adverse yaw was obtained while elimination of both angle and roll rate feedback terms resulted in a 17% increase in measured adverse yaw. The resulting state trajectories were similar to those obtained for the optimal control discussed previously. The increased adverse yaw responses were due to the reduced controller magnitude commanded by the lesser number of feedback states. The control designer should be able to produce similar minimum adverse yaw conditions to those reduced number of feedback terms by increasing the state weighting matrix terms and recomputing the Riccati solution.

Summary

Optimal control techniques have been presented for controlling the buildup of adverse yawing motion during lateral offset maneuvers in the final stages of an aircraft landing approach. The controllers also act to control sideslip disturbances and the associated lateral-directional coupled motions. The control process developed indicated that as much rudder control as considered safe, from emergency control failure considerations, should be utilized in conjunction with lateral control to achieve the optimum minimum adverse yaw condition.

In conclusion the following has been determined. 1) The linearized equations of motion adequately predict aircraft lateral response during large bank angle rolling maneuvers in the landing approach environment. 2) Optimally controlled control systems, utilizing constant-gain state variable feedback, adequately control the adverse yaw generated during these rolling maneuvers within acceptable limits. 3) The optimum control system for adverse yaw is obtained by utilizing the maximum rudder considered safe, from emergency control failure considerations coupled with a reduction in the pilot's commanded lateral control.

References

[1]Griggs, S. D., Branch, W. M., and Bischoff, D. E., "Phase I, Navy Preliminary Evaluation of the EA-6B Airplane," Rept. FT-21R-69, March 1969, Naval Air Test Center, Patuxent River, Md.

[2]Kirk, D. E., "*Optimal Control Theory*," Prentice Hall, New York, 1970.

[3]Langdon, S. D., "*Fixed Wing Stability and Control Theory and Flight Test Techniques*," Aug. 1969, Naval Air Test Center, Patuxent River, Md.